TRINITY COLLEGE

YEAR 12
MATHEMATICS
SPECIALIST

Test 2, 2023
Section One: Calculator Free
Vectors in 3D

STUDENT'S NAME: _____

DATE: Wednesday 10th May **TIME:** 16 minutes **MARKS**: 16
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items:

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

This page has been intentionally left blank.

Question 1 **(10 marks)**

Consider the points $A = (1, 4, 7)$ $B = (-2, 4, 10)$ $C = (3, 4, 2)$ $D = (4, 0, 7)$

a) Determine the equation of the plane containing points A, B, C in the form $\boldsymbol{r}.\boldsymbol{n} = k$ (3 marks)

b) Determine the equation of the line going through points C and D. (2 marks)

c) Determine an un-simplified expression for the angle between the plane and the line found in parts a) and b) respectively. (2 marks)

d) Determine the equation of the sphere that has $\overrightarrow{AD}$ as its diameter. (3 marks)

Question 2 **(6 marks)**

Consider the following system of equations

$$\begin{aligned} 3x - 4y + z &= 10 \\ 6x - 8y + kz &= 20, \quad \text{where } k \in \mathbb{R} \\ 2x + 8y - 2z &= 4 \end{aligned}$$

a) If $k = 2$, describe the type of solution the system produces and state the geometric nature of the situation. (*Note, do not find the solution*). (2 marks)

b) If $k = 1$,

i) solve the system of equations. (3 marks)

ii) State the geometric interpretation of your result to b i). (1 mark)

END OF QUESTIONS

TRINITY COLLEGE

YEAR 12
MATHEMATICS
SPECIALIST

Test 2, 2023
Section Two: Calculator Allowed
Vectors in 3D

STUDENT'S NAME: _____

DATE: Wednesday 10th May **TIME:** 35 minutes **MARKS**: 32
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

This page has been intentionally left blank.

Question 3 **(9 marks)**

For this question, assume gravity does not exist – This is not a projectile motion question. Also assume that the tip of the arrow is being defined by the vector function.

An archer, A, fired an arrow with the following vector function representing the arrow's position, where t represents seconds after being fired.

$$\overrightarrow{OA_t} = \begin{bmatrix} 5+t \\ 5-2t \\ 5-0.5t^2 \end{bmatrix}$$

A spherical balloon has the vector equation $\left| \boldsymbol{r} - \begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix} \right| = 1$.

a) Determine the cartesian equation of this spherical balloon. (1 mark)

b) The arrow first hit the balloon 1.654 seconds after being fired. Show how this time value was calculated, and state the time solution to 4 decimal places. (3 marks)

c) Calculate the co-ordinate where the arrow first hit the balloon. (1 mark)

d) Determine the speed of the arrow when it first hit the balloon. (2 marks)

e) Determine the total distance travelled from when it was fired to when it first hit the balloon.

The distance formula is $\int_a^b |\boldsymbol{v}(t)|dt$ (2 marks)

Question 4 **(10 marks)**

On the moon, gravity accelerates all things downward at $1.62m/s^2$.

A machine can launch rocks at $5m/s$ at an angle of $\theta°$ to the positive x axis from the origin.

a) Show how the displacement of the rock after t seconds can be shown by
$\boldsymbol{r} = \begin{bmatrix} 5\cos(\theta)\, t \\ -0.81t^2 + 5\sin(\theta)\, t \end{bmatrix}$ meters (3 marks)

On a flat ground, a rock was launched at $56°$ to the positive x axis.

b) Calculate when the rock was at its highest point. (2 marks)

c) Calculate the horizontal distance the rock travelled when it first hit the ground. (3 marks)

d) Using the fact that fMax$(\sin(\alpha)\cos(\alpha))$ occurs when $\alpha = 45°$, prove that launching at an angle of $45°$ to the positive *x* axis will lead to the rock travelling the furthest horizontal distance.
(2 marks)

Question 5 **(8 marks)**

A hydraulic press can create a huge crushing force between two flat surfaces.
Typically, the bottom plate is fixed whilst the top plate moves.

Consider two parallel planes which represent the two parallel surfaces of the hydraulic press:

$$r.\begin{bmatrix}3\\5\\6\end{bmatrix} = 2 \quad \text{and} \quad r.\begin{bmatrix}6\\10\\12\end{bmatrix} = k, \quad k \geq 4$$

a) Explain mathematically why these two surfaces are parallel, and explain which plane represents the top, moving plate. (1 mark)

A spherical marble of diameter 10cm is trapped between the plates but has not yet been crushed.

b) Determine any possible vector equation for a sphere representing a marble of diameter 10cm trapped between the two planes. (4 marks)

c) Determine the value of k. (3 marks)

Question 6 **(5 marks)**

A fly is travelling with a position vector of $OF_t = \begin{bmatrix} 3+2t \\ -3+t \\ 7+0.5t \end{bmatrix}$ and a wasp is hunting it with a position vector of $OW_t = \begin{bmatrix} 8+1.2t \\ 15-1.88t \\ 4+ae^t \end{bmatrix}$.

a) If there is a collision, determine a and the time when the collision occurs. (2 marks)

b) Determine the initial velocity of the wasp. (1 mark)

When hunting, a dragonfly predicts the future location of it's target and flies in a straight line to collide, travelling at a constant $3m/s$.

(c) A dragonfly is also hunting the fly. If the dragonfly starts at the location (1, 5, 3), determine a system of 4 linear equations that would be solved to find the velocity of the dragonfly and the time of collision between the dragonfly and the fly.
DO NOT SOLVE THE EQUATIONS. (2 marks)

END OF QUESTIONS